Birzeit University
Faculty of Engineering
Department of Electrical Engineering
Engineering Probability and Statistics ENEE 2309
Problem Set (1)
Fundamental Concepts of Probability

1) Let *A* and *B* denote two events defined over a sample space *S*. Suppose it is given that P(AUB) = 0.76

a. If A and B are mutually exclusive events with $P[\overline{A}] = 0.45$, then what is P[B]?
b. Suppose, instead, that A and B are independent events with $P[B/A] = 0.12$,. What is $P[A]$?
c. If it is given that $P[A] = 0.30$ and that $P[B/A] = 0.60$, what is P[B]?

2) In a certain lot of personal computers, it is known that 1 % have some minor defect as they come off the production line. They are put through a test procedure, which detects any defect 98 % of the time if a defect is really present, and indicates a defect 1 % of the time even though there is none present. What is the probability that

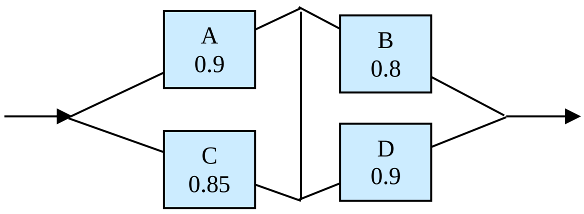
a. a computer will be classified defective as a result of the test procedure?
b. a computer is in fact defective if the test indicates that it is defective?

3) A sample space consists of three events, A, B and C. If $P(A^c) = 0.5$, and $P(A \cap B) = 0.25$, P(BUC) = 0.75. The pair of events (A and B), (B and C) are independent. Events A and C are mutually exclusive. Find the followings:

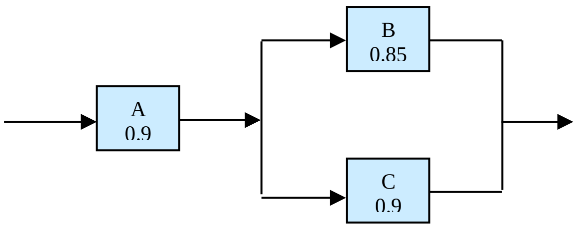
a. Probability that exactly one event will occur.
b. P(B/A)

4) Consider the following systems made up of independent components. The probability that each component functions is indicated in the figures.

a. Find the probability that the systems work properly (system reliability).
b. Is it possible to increase the reliability up to 99.5% for the system in (b) by adding more components in the parallel connection?



(a)



(b)

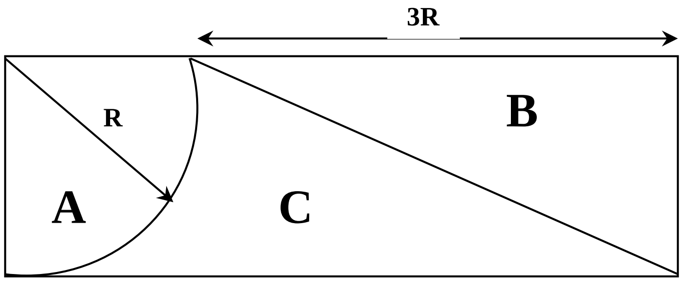
5) Items in a production line have to pass two successive quality control tests. If the probability of producing items of high quality is 0.95, the probability of misclassifying the items through the first and second tests is 0.05 and 0.02 respectively.

a. Find the percentage of items classified as high quality.

b. If an item is classified as high quality, what is the probability that it came out of the production line as high quality?

6) An irrigation well is to be drilled in the area shown in the figure. The probability of obtaining water for each sub-area A, B and C is 0.8, 0.1 and 0.3 respectively. The probability of selecting any of the three sub-areas is proportional to the area

a. Find the probability of obtaining water.
b. If no water was obtained, what is the probability that the well was drilled in area C?



7) An urn contains colored balls: 4 Red balls and 6 Green balls. Suppose that three balls are drawn from this urn.

a. If the three balls are drawn one after another without replacement, what is the probability that the colors observed will be Red, Green in this order?
b. If the three balls are drawn one after another with replacement, what is the probability that all three of the selected balls will be of the same color?
c. If the three balls are drawn simultaneously from this urn (thus without replacement), what is the probability that the selected balls will all be of the same colors?

8) Prove that a set with n elements has 2^n subsets.

9) For a storm-sewer system, estimates of annual maximum flow rates (AMFR) and their likelihood of occurrence [assuming that a maximum of 12 cubic feet per second, cfs, is possible] are given as:

Event A: ($5 \leq$ AMFR ≤ 10 cfs)	P(A) = 0.6
Event B: ($8 \leq$ AMFR ≤ 12 cfs)	P(B) = 0.6
Event $C=A\cup B$	P(C) = 0.7

Determine P ($8 \leq$ AMFR ≤ 10 cfs), the probability that AMFR is between 8 and 10 cfs.

10) Given that P(A) = P(B) = P(C) = ¼, $P(A\cap B) = P(C\cap B) = 0$, $P(A\cap C) = 1/8$, find

a. the probability that at least one of the events A, B, or C occurs
b. P(AUBUC)
c. the probability that exactly one even occurs

11) A box contains three coins. One coin is two-headed, a second is fair, and the third is biased with p being the probability of getting a head. A coin is chosen at random from the box and flipped once

a. What is the probability that the flip results in a head?
b. Suppose that the flip yields a head, what is the probability that the chosen coin is the biased one?

12) An experiment is independently repeated five times. The probability of a success in every trial is 0.5, find the probability of obtaining three consecutive successes.

13) An experiment is independently repeated until a success is obtained for the first time. What is the probability that it will take five trials for that to happen assuming that the probability of a success in each trail is p.

14) A motor drives an electric generator. During a 30-day period, the motor needs repair with probability 8% and the generator, independently of the motor, needs repair with probability 4%. What is the probability that during the given period, the entire apparatus will need repair?

15) Assume that the reaction time of a driver over the age of 70 to a certain visual stimulus is described by a continuous probability function of the form $f(x) = xe^{-x}$, x>0, where x is measured in seconds. Let A be the event "Driver requires longer than 1.5 seconds to react". Find P(A).

16) One model to describe the mortality is $f(t) = kt^2(100-t)^2$, $0 \le t \le 100$, where t describes the age at which a person dies.

a. Find k
b. Let A be the event "Person lives over 60". Find P(A)
c. What is the probability that a person will die between the age of 80 and 85 given that the person has lived to be at least 70?

17) One integer is chosen at random from the numbers {1, 2, ……, 10}. What is the probability that the chosen number is divisible by 6? Assume all 10 outcomes are equally likely.

18) One integer is chosen at random from the numbers {1, 2, ……, 10}. Assume the probability of occurrence of an even number is twice as likely as that of an odd number. What is the probability that the chosen number is divisible by 6?

19) One integer is chosen at random from the numbers {1, 2, ……, 6}. Assume the probability of occurrence of any outcome is directly proportional to its value. What is the probability that the chosen number is divisible by 6?

20) Suppose we roll two fair dice (حجري نرد) - a red die and a green die. Let A = "at least one of the dice shows a 2 or a 5", B = "The sum of the two dice is between 8 and 10 (inclusive)".

a) Compute P(A).
b) Compute P(B).
c) Are events A and B mutually exclusive? (Justify!)
d) Compute $P(A \cup B)$.(Show your work)
e) Are events A and B independent? (Justify!)
f) Compute $P(A/B)$.(Show your work)

21) Suppose we roll two fair dice a red die and a green die. Let A = "The red die shows a 2 or a 5", B = "The sum of the two dice is between 8 and 10 (inclusive)".

a) Compute P(A).

b) Compute P(B).
c) Are events A and B mutually exclusive? (Justify!)
d) Compute $P(A \cup B)$.(Show your work)
e) Are events A and B independent? (Justify!)
f) Compute $P(A/B)$.(Show your work)

22) In a certain assembly plant, three machines, B_1, B_2, and B_3, make 30%, 45%, and 25%, respectively, of the products. It is known from past experience that 2%, 3%, and 2% of the products made by each machine, respectively, are defective. Now, suppose that a finished product is randomly selected. What is the probability that it is defective?

23) A manufacturing firm employs three analytical plans for the design and development of a particular product. For cost reasons, all three are used at varying times. In fact, plans 1, 2, and 3 are used for 30%, 20%, and 50% of the products, respectively. The defect rate is different for the three procedures as follows: $P(D|P_1) = 0.01$, $P(D|P_2) = 0.03$, $P(D|P_3) = 0.02$, where $P(D|P_j)$ is the probability of a defective product, given plan j. If a random product was observed and found to be defective, which plan was most likely used and thus responsible?

24) A factory has two production lines **A** and **B**, production line **A** works 7 days a week, production line **B** works only 5 days a week. Production line A produces 5000 items each day where 90% of the produced items are high quality, and 10% are medium quality. Production line **B** produces 3500 items each day where 80% of the produced items are high quality, 10% medium quality, and 10% of low quality.

a) What is the probability that a randomly selected item produced by production line **A** is of high quality?
b) What is the probability that a randomly selected item produced by production line **B** is of medium quality?
c) What is the probability of selecting a medium quality item produced by production line **B**?
d) What is the probability that a low quality item is produced?
e) What is the probability that a high quality item is produced?
f) If an item is tested and found to be of medium quality, what is the probability that it was produced by production line **A**.
g) If an item is tested and found to be of low quality, what is the probability that it was produced by production line **B**.